

# Conformal General Relativity

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## Abstract

The inflation-free solution of problems of the modern cosmology (horizon, cosmic initial data, Planck era, arrow of time, singularity, homogeneity, and so on) is considered in the conformal-invariant unified theory given in the space with geometry of similarity where we can measure only the conformal-invariant ratio of all quantities. Conformal General Relativity is defined as the  $SU_c(3) \times SU(2) \times U(1)$ -Standard Model where the dimensional parameter in the Higgs potential is replaced by a dilaton scalar field described by the negative Penrose-Chernikov-Tagirov action. Spontaneous  $SU(2)$  symmetry breaking is made on the level of the conformal-invariant angle of the dilaton-Higgs mixing, and it allows us to keep the structure of Einstein's theory with the equivalence principle. We show that the lowest order of the linearized equations of motion solves the problems mentioned above and describes the Cold Universe Scenario with the constant temperature  $T$  and  $z$ -history of all masses with respect to an observable conformal time. A new fact is the intensive cosmic creation of  $W, Z$ - vector bosons due to their mass singularity. In the rigid state, this effect is determined by the integral of motion  $(m_w^2 H_{\text{hubble}})^{1/3} = 2.7 K k_B$  that coincides with the CMB temperature and has the meaning of the primordial Hubble parameter. The created bosons are enough to consider their decay as an origin of the CMB radiation and all observational matter with the observational element abundances and the baryon asymmetry. Recent Supernova data on the relation between the luminosity distance and redshift (including the point  $z = 1.7$ ) do not contradict the dominance of the rigid state of the dark matter in the Conformal Cosmology.

## 1. Statement of the Problem

We would like to present here the results obtained by our international group on the construction of a unified theory of all interactions based on the principle of relativity of all standards of measurement [1]-[6]. This principle can be incorporated into the unified theory through the Weyl geometry of similarity as a manifold of conformal-equivalent Riemannian geometries. To escape defects of the first Weyl version of 1918 [7], we use the scalar-tensor conformal invariant ( $\hat{g}_{\mu\nu} = W^2 g_{\mu\nu}$ ) where  $W$  is a dilaton scalar field described by the negative Penrose-Chernikov-Tagirov (PCT) action [8]

$$S = - \int d^4x \sqrt{-\hat{g}} \frac{1}{6} R(\hat{g}) .$$

This action keeps the structure of Einstein's theory and is a conformal analog of the Einstein-Hilbert action. Therefore, we call this theory the Conformal General Relativity (CGR).

In contrast to Einstein's General Relativity, we can measure only a ratio of two Einstein intervals that depends only on nine components of the metric tensor. This means that

the conformal invariance allows us to remove only one component of the metric tensor but not the dilaton. The best way to remove this component for the Hamiltonian description is to use the scale-free Lichnerowicz conformal-invariant field variables  $F_{(n)}$ , including the metric  $g$  [9]

$$F_{(n)}^L = ||^{(3)}g||^{-n/6} F_{(n)} , \quad (ds)_L^2 = g_{\mu\nu}^L dx^\mu dx^\nu, \quad ||^{(3)}g^L|| = 1 ,$$

where  $(n)$  is the conformal weight for a tensor ( $n = 2$ ), vector ( $n = 0$ ), spinor ( $n = -3/2$ ), and scalar ( $n = -1$ ). We show how the conformal invariance of the action, variables, and measurable quantities allows us to solve the problems of modern cosmology without inflation.

After the formulation of the theory and a method, we discuss the Cold Universe Scenario where the Universe begins to form the intensive cosmic creation of  $W, Z$ -vector bosons with the constant temperature  $T = (m_w^2 H_{\text{hubble}})^{1/3} = 2.7K$  as one of the integrals of motion [6].

## 2. Theory

Conformal General Relativity (CGR) is defined as the  $SU_c(3) \times SU(2) \times U(1)$ -Standard Model (SM) where the dimensional parameter in the Higgs potential is replaced by the dilaton  $W$  described by the PCT-action [6] so that

$$\mathcal{L}_{\text{Higgs}} = -\lambda \left[ (|\Phi|)^2 - y_h^2 W^2 \right]^2 .$$

The conformal-invariant interactions of the dilaton and the Higgs doublet form the effective Newton coupling in the gravitational Lagrangian

$$\frac{|\Phi|^2 - W^2}{6} R .$$

This coupling shows a necessity of the dilaton-Higgs mixing [10]

$$W = \phi \cosh \chi, \quad |\Phi| = \phi \sinh \chi \quad (|\Phi|^2 - W^2 = -\phi^2) ,$$

so that the CGR action takes the form

$$\mathcal{L}_{\text{CGR}} = -\frac{\phi^2}{6} R - \partial_\mu \phi \partial^\mu \phi + \phi^2 \partial_\mu \chi \partial^\mu \chi + \mathcal{L}_{\text{Higgs}} + y_e \phi \sinh \chi \bar{e} e + \dots ,$$

where the Higgs Lagrangian

$$\mathcal{L}_{\text{Higgs}} = -\lambda \phi^4 \left[ \sinh^2 \chi - y_h^2 \cosh^2 \chi \right]^2$$

describes the conformal-invariant Higgs effect of the spontaneous  $SU(2)$  symmetry breaking

$$\frac{\partial \mathcal{L}_{\text{Higgs}}}{\partial \chi} = 0 \Rightarrow \chi_1 = 0, \quad |\sinh \chi_2| = \frac{y_h}{\sqrt{1 - y_h^2}} \sim 10^{-17} .$$

This effect is made on the level of the mixing angle, and it takes place even for  $\lambda = 0$ . In this case, the trivial solution  $\chi = \text{constant}$  leads to the Higgs particle free unified

model [2], where any measurable ratio of masses at the same point is a constant, and the equivalence principle is fulfilled.

The present-day value of the dilaton in the region far from heavy masses distinguishes the scale of the Planck mass

$$\phi(t_0, x) \simeq M_{\text{Planck}} \sqrt{\frac{3}{8\pi}} .$$

This fact is revealed by the energy-constrained perturbation theory [1]-[4].

### 3. Method

The lowest order of the energy-constrained perturbation theory is formed by linearization of all equations of motion in the class of functions with nonzero Fourier harmonics (i.e. the "local" class of functions) in the flat conformal space-time

$$ds_L^2 = d\eta^2 - dx_i^2 , \quad d\eta = N_0(t)dt , \quad N_0 = [g_L^{00}]^{-1/2} . \quad (1)$$

Part of these local equations are constraints that form the projection operators. These operators remove all superfluous degrees of freedom of massless and massive local fields. In particular, four local constraints as the equations for  $g_{\mu\nu=0}$  remove three longitudinal components of gravitons and all nonzero Fourier harmonics of the dilaton. However, the local constraints could not remove the zero Fourier component of the dilaton

$$\phi^L(t, x) = \varphi(t) .$$

The infrared interaction of the complete set of local independent variables  $\{f\}$  with this dilaton zero mode  $\varphi(t)$  is taken into account exactly. The lowest order of the considered linearized perturbation theory is described by the Hamiltonian form of the CGR action in this approximation

$$S_0 = \int_{t_1}^{t_2} dt \int_{V_0} d^3x \left( \sum_f p_f \dot{f} - P_\varphi \dot{\varphi} - N_0 \left[ -\frac{P_\varphi^2}{4} + \rho(\varphi) \right] \right) ,$$

where  $\rho(\varphi)$  is the global energy density that generates all the above-mentioned linear equations for independent degrees of freedom. This energy-constrained theory contains the Friedmann-like equation for the conformal time (1)

$$\eta(\varphi_0, \varphi_I) = \pm \int_{\varphi_I}^{\varphi_0} \frac{d\varphi}{\sqrt{\rho(\varphi)}} \quad (2)$$

as a consequence of the energy constraint

$$-\frac{P_\varphi^2}{4} + \rho_F = 0$$

and the equation for the dilaton momentum  $P_\varphi$

$$\frac{d\varphi}{d\eta} = \frac{P_\varphi}{2} = \pm \sqrt{\rho(\varphi)} .$$

The cosmic evolution of dilaton masses leads to the redshift of energy levels of star atoms [11] with the energy density  $\rho(\varphi)$  and the Hubble parameter  $H_0 = \frac{\varphi'}{\varphi}(\eta_0)$ , which gives the present day value of the dilaton

$$\varphi(\eta_0) = \frac{\sqrt{\rho_0}}{H_0} = \Omega_0^{1/2} M_{\text{Planck}} \sqrt{\frac{3}{8\pi}} .$$

Therefore, the Planck scale is distinguished as a current (present-day) value of the dilaton, rather than the fundamental parameter that can be shifted into the beginning of the Universe.

The energy-constrained theory solves also the problems of homogeneity and horizon (by the perturbation theory in the conformal space (1)), the positive arrow of conformal time (2) (by the Dirac quantization of the dilaton motion forward with a positive energy that we call a universe, and, backward with a negative one that we call an anti-universe), a cosmic singularity (by nonzero boundary conditions for the dilaton in the observable Universe), the Higgs particle, monopoles, and walls (by the consideration of the Higgs potential-free version), the cosmic initial data (by the diagonalization of the energy density and the equations of motion) [4, 6].

The energy density can be represented in the diagonal form

$$\rho(\varphi) = \sum_{\varsigma} \omega_f(\varphi, k) \hat{N}_{\varsigma}$$

(where  $\omega_f(\varphi, k) = \sqrt{k^2 + y_f^2 \varphi^2}$  is the one-particle energy;  $\hat{N}_{\varsigma} = \frac{1}{2}(a_{\varsigma}^+ a_{\varsigma} + a_{\varsigma} a_{\varsigma}^+)$  is the number of particles;  $\varsigma$  include momenta  $k_i$ , species  $f = h, \gamma, v, s, \chi$ , spins  $\sigma$ ), if we introduce "particles" as the holomorphic field variables

$$f(t, \vec{x}) = \sum_k \frac{C_f(\varphi) \exp(ik_i x_i)}{V_0^{3/2} \sqrt{\omega_f(\varphi, k)}} \frac{1}{\sqrt{2}} \left( a_{\sigma}^+(-k, t) \epsilon_{\sigma}(-k) + a_{\sigma}(k, t) \epsilon_{\sigma}(k) \right) ,$$

where

$$C_{\chi}(\varphi) = \frac{\sqrt{2}}{\varphi}, \quad C_h(\varphi) = \frac{\sqrt{12}}{\varphi}, \quad C_{(f=\gamma, s)}(\varphi) = 1, \quad C_v^{\perp} = 1, \quad C_v^{\parallel} = \frac{\omega_v}{y_v \varphi} .$$

At the same time, the canonic differential form in the action acquires nondiagonal terms as sources of cosmic creation of particles

$$\left[ \int_{V_0} d^3x \sum_{f, k} p_f \dot{f} \right]_B = \sum_{\varsigma=(k, f, \sigma)} \frac{\imath}{2} (a_{\varsigma}^+ \dot{a}_{\varsigma} - a_{\varsigma} \dot{a}_{\varsigma}^+) - \sum_{\varsigma} \left( \frac{\imath}{2} (a_{\varsigma}^+ a_{\varsigma}^+ - a_{\varsigma} a_{\varsigma}) \right) \dot{\Delta}_{\varsigma}(\varphi) .$$

The number of created particles is calculated by the diagonalization of equations of motion by the Bogoliubov transformation

$$b_{\varsigma}^+ = \cosh(r_{\varsigma}) e^{-i\theta_{\varsigma}} a_{\varsigma}^+ - \imath \sinh(r_{\varsigma}) e^{i\theta_{\varsigma}} a_{\varsigma},$$

$$b_{\varsigma} = \cosh(r_{\varsigma}) e^{i\theta_{\varsigma}} a_{\varsigma} + \imath \sinh(r_{\varsigma}) e^{-i\theta_{\varsigma}} a_{\varsigma}^+$$

The equations for Bogoliubov coefficients

$$[\omega_\varsigma - \theta'_\varsigma] \sinh(2r_\varsigma) = \Delta'_\varsigma \cos(2\theta_\varsigma) \cosh(2r_\varsigma),$$

$$r'_\varsigma = -\Delta'_\varsigma \sin(2\theta_\varsigma)$$

determine the number of particles

$$\mathcal{N}_\varsigma^{(B)}(\eta) = {}_{sq} \langle 0 | \hat{N}_\varsigma^{(B)} | 0 \rangle_{sq} - \frac{1}{2} = \sinh^2 r_\varsigma(\eta)$$

created during the time  $\eta$  from squeezed vacuum:  $b_\varsigma |0\rangle_{sq} = 0$  and the evolution of the density

$$\rho(\varphi) = \varphi'^2 = \sum_\varsigma \omega_\varsigma(\varphi) {}_{sq} \langle 0 | \hat{N}_\varsigma | 0 \rangle_{sq} .$$

The set of nondiagonal terms in SM

$$\begin{aligned} \Delta_h(\varphi) &= \ln(\varphi/\varphi_I), \\ \Delta_v^\perp(\varphi) &= \frac{1}{2} \ln(\omega_v/\omega_I) , \\ \Delta_v^\parallel(\varphi) &= \Delta_h(\varphi) - \Delta_v^\perp(\varphi) , \\ \Delta_\chi(\varphi) &= \Delta_h(\varphi) + \Delta_v^\perp(\varphi) , \end{aligned}$$

where  $\varphi_I$  and  $\omega_I$  are initial data, contains the zero-mass singularity [12, 13] that plays an important role in the primordial creation of longitudinal vector bosons with the properties of the cosmic microwave background (CMB) radiation.

## 4. Results

In the limit of the Early Universe  $\varphi \Rightarrow 0$ , the CGR action gives the well-known rigid state  $\rho/\rho_0 = \Omega_{\text{rigid}}(z+1)^2$  and the primordial motion of the dilaton

$$(\varphi^2)'' = 0 \Rightarrow \varphi^2(\eta) = \varphi_I^2[1 + 2H_I\eta] = \frac{\varphi_0^2}{(1+z)^2} , \quad H(z) = \frac{\varphi'}{\varphi} = H_0(1+z)^2 .$$

At the point of coincidence of the Hubble parameter of this motion with the mass of vector bosons  $m_v(z) \sim H(z)$ , there occurs the intensive creation of longitudinal bosons (see Fig.1).

The temperature of thermal equilibrium of bosons can be estimated from the restriction for the inverse time of relaxation  $\eta_{\text{relaxation}}^{-1} = \sigma_{\text{scat.}} n_v(T_{eq}) \geq H(z)$ , and it is the integral of motion  $T_{eq} \simeq [m_v^2(z)H(z)]^{1/3} \simeq (m_W^2 H_0)^{1/3} = 2.7K \sim H_I$ . It is wonderful that the present-day value of the boson mass and Hubble parameter gives the Cosmic Microwave Background temperature.

The opposite (present-day stage) limit  $\varphi \Rightarrow M_{\text{Planck}}\sqrt{\frac{3}{8\pi}}$  includes the dust stage  $\rho/\rho_0 \sim \Omega_M/(z+1)$  with the accelerating evolution in the conformal time. As a light ray traces a null geodesic that satisfies the equation  $dr/d\eta = 1$ , the coordinate distance as a function of the redshift  $z$  in the Conformal Cosmology (CC)

$$H_0 r(z) = \int_1^{1+z} \frac{dx}{\sqrt{\Omega_{\text{Rid.}} x^6 + \Omega_{\text{Rad.}} x^4 + \Omega_M x^3 + \Omega_\Lambda}} \quad \left( \sum_{I=\text{Rid.}, \text{Rad.}, \text{M}, \Lambda} \Omega_I = 1 \right)$$

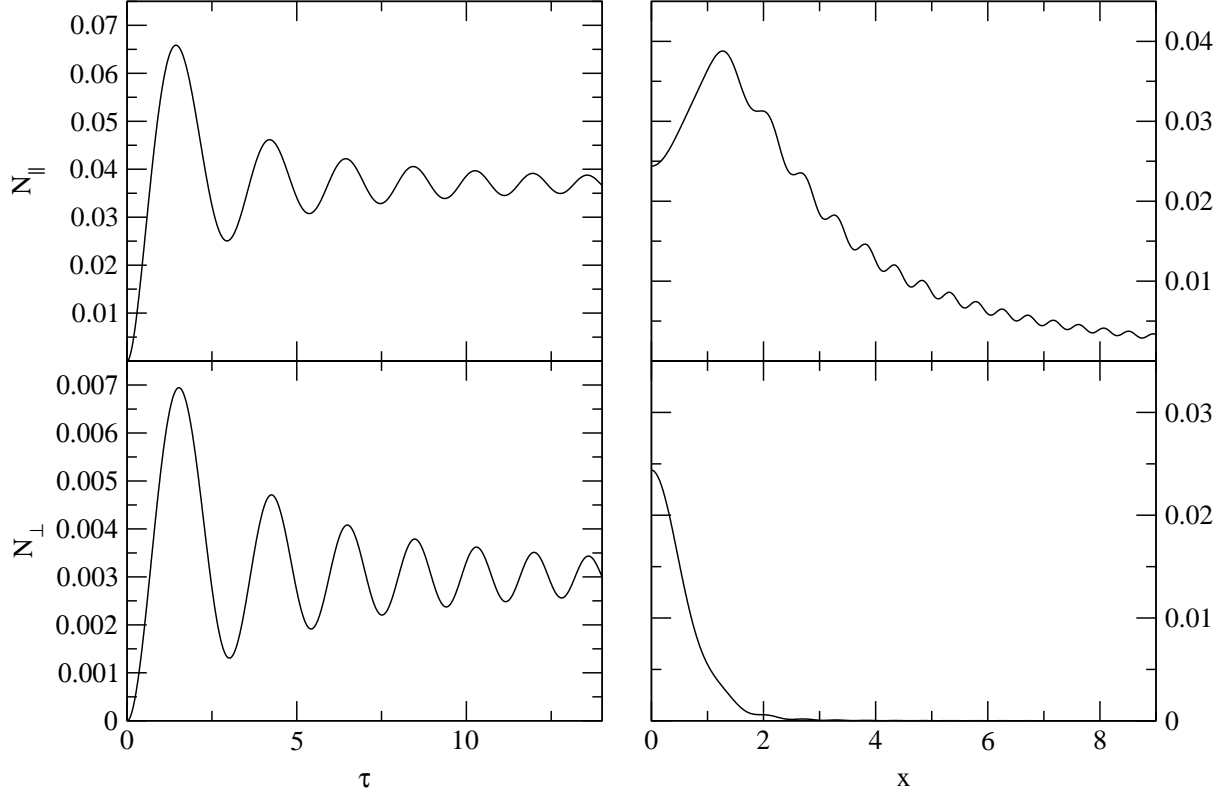


Figure 1: Time dependence for the dimensionless momentum  $x = k/H_I = 1.25$  (left panels) and momentum dependence, at the dimensionless lifetime  $\tau = (\eta 2 H_I) = 14$ , (right panels) of the transverse (lower panels) and longitudinal (upper panels) components of the vector-boson distribution function [6]

coincides with the similar relation between coordinate distance and redshift in Standard Cosmology (SC).

The luminosity distance  $\ell$  is defined so that the apparent luminosity of any object behaves as  $1/\ell^2$ . Therefore, in comparison with the stationary space in SC and stationary masses in CC, a part of photons is lost. To restore the full luminosity in both SC and CC, we should multiply the coordinate distance by the factor  $(1+z)^2$ . This factor comes from the evolution of the angular size of the light cone of emitted photons in SC and from the increase of the angular size of the light cone of absorbed photons in CC. However, in SC, we have an additional factor  $(1+z)$  due to the expansion of the universe, since measurable distances in SC are related to measurable distances in CC (that coincide with the coordinate ones) by the relation

$$\ell = a \int \frac{dt}{a} = ar(z), \quad a = \frac{\varphi}{\varphi_0} = \frac{1}{1+z} . \quad (3)$$

Thus we obtain the relations

$$\ell_{SC}(z) = (1+z)^2 \ell = (1+z)r(z) ,$$

$$\ell_{CC}(z) = (1+z)^2 r(z) .$$

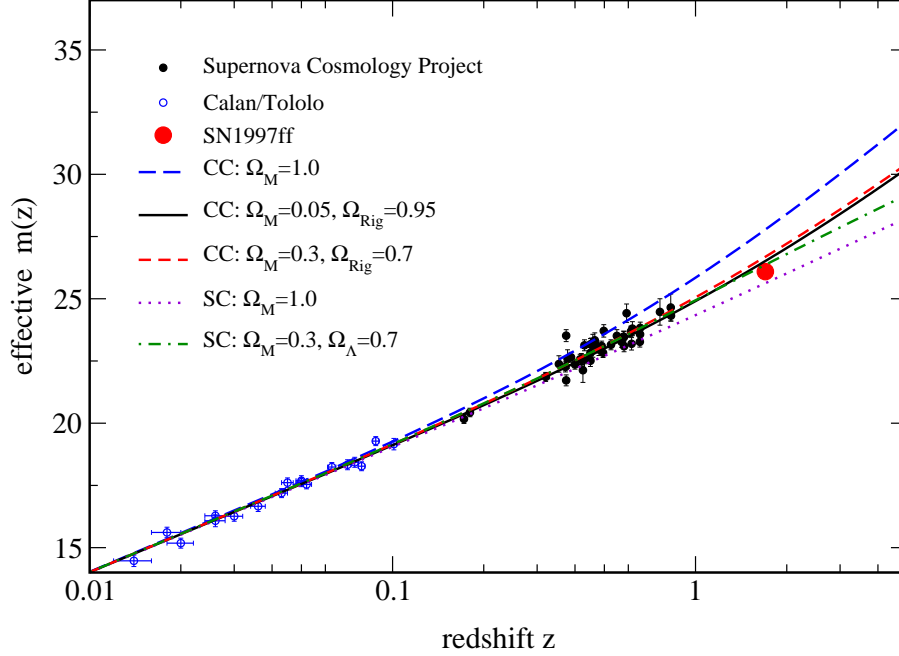


Figure 2:  $m(z)$ - relation for a flat universe model in SC and CC. The data points include those from 42 high-redshift Type Ia supernovae [14] and those of recently reported farthest supernova SN1997ff [15]. An optimal fit to these data within the SC requires the cosmological constant  $\Omega_\Lambda = 0.7$ , whereas in the CC presented here no cosmological constant is needed.

This means that the observational data are described by different regimes in SC and CC. In Fig. 2, we compare the results of SC and CC for the relation between the effective magnitude and redshift:  $m(z) = 5 \log [H_0 \ell(z)] + M$  where  $M$  is a constant with recent experimental data for distant supernovae.

There is also nonzero baryon asymmetry due to the squeezed vacuum expectation value of the winding number functional of the primordial transversal bosons for their lifetime, if we have three Sakharov conditions: 1)  $CP_{SM}$ , 2)  $H_0 \neq 0$ , 3)  $\Delta L = 3\Delta B = \Delta n_w + \Delta n_z \neq 0$

$$\Delta n_{w,z} = \frac{\alpha_{w,z}}{4\pi} \int_0^{\eta_1^{w,z}} d\eta \int d^3x \quad {}_{sq} \langle 0 | E_i^W B_i^W | 0 \rangle {}_{sq},$$

$$\alpha_w = \frac{4\alpha_{QED}}{\sin^2 \theta_W}, \quad \alpha_z = \frac{\alpha_{QED}}{\sin^2 \theta_W \cos^2 \theta_W}, \quad \eta_l^w = 15H_I^{-1}, \quad \eta_l^z = 30H_I^{-1}.$$

Thus, we can propose the Cold Universe Scenario with the  $z$ -history of masses and the invariant temperature  $\sim 2.7K$ . The Universe begins from the rigid primordial motion of the modulus of the dilaton-Higgs mixing. This motion creates vector bosons, the decay of

which forms all matter in the Universe with the constant temperature that coincides with the primordial Hubble parameter. There are arguments in favor of that the Cold Universe Scenario reproduces all results of the Hot one on the primordial element abundances in the radiation stage, since we have in CC the same (square root) dependence of the scale factor (3) on the observable time in the rigid stage and the same argument for the Boltzmann factors. In contrast to SC, this rigid stage dynamics of the chemical evolution in CC does not contradict recent Supernova data.

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